Topology

M. Math. I

Mid-Term Examination

Instructions: All questions carry equal marks.

1. Let X and Y be spaces and $A \subset X$ be a subspace. Let $f : A \to Y$ be a continuous map. If Y is Hausdorff, then show that there exists at most one extension of f to the closure $\overline{A}$ of A. Give an example to show that this statement is false if Y is not Hausdorff. Justify your answer.

2. Let $X_n = [0, \frac{1}{n}]$. If $\prod X_n$ is given the product topology, then prove that every continuous real valued function $f : \prod X_n \to \mathbb{R}$ is bounded. Give an example of an unbounded continuous real valued function on $\prod X_n$ when it is endowed with the box topology. Justify your answer.

3. Define linear continuum. Let X be an ordered set. If X is connected with respect to the order topology, then prove that X is a linear continuum.

4. Let X be a connected metric space. Then prove that either X is a singleton set or contains uncountable number of points. Give an example of a connected topological space with countable number of points.

5. Let X and Y be topological spaces and $\mathcal{A}$ be a collection of basic open sets in $X \times Y$ such that no finite subcollection of $\mathcal{A}$ covers $X \times Y$. If X is compact, show that there exists a point $x \in X$ such that no finite subcollection of $\mathcal{A}$ covers $\{x\} \times Y$.

6. Define a regular topological space. Prove that if a space X is regular, then any two distinct points of X have open neighbourhoods whose closures are disjoint.